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## Transfer by a Manipulator with a Three-finger Grasp of a Brittle Cylinder<sup>1</sup>

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We consider the problem of the brittle cylinder grasping by the  $n$  fingers of the robot-manipulator. Each finger contacts the cylinder in a single supporting point with Amontons-Coulomb or for two footholds spinning friction. Using numerical simulations and analytically, possible locations of contact points on the cylinder, for which there is a kinetostatics problem solution when the cylinder is moved by three fingers, are received. By the analogy of the equilibrium of a three-legged robot on a cylinder for the problems of transfer by a manipulator with a three-finger grasp of a cylinder or for a robot on a surface which legs suspension points are on a cylinder surface. Two supporting points can be on one diameter in the cylinder base. Or because of friction on the opposite sides of the robot center of mass or giving in the dynamics, it is point  $C$ . The analogy of the problem is oscillations in the vicinity of the stable equilibrium one cylinder on another. The cylinder lies on one finger rectangular to it, of the hand of a humanoid robot, adheres to the end of the other finger. Similarly holds the glass. Robot can hold the horizontal cylinder by three fingers. Let one of the points is in vertical plane containing cylinder axis and another are in the plane orthogonal to the axis. The supporting points are on the external surface of the lower semi-cylinder and the cylinder center mass is in the footholds triangle. The supporting set is divided into two subsets.

**Keywords:** three-finger grasp, Amontons-Coulomb friction, three-legged robot

### 1. A cylinder grasping problem

In this paper, we consider the problem of curved object grasping by the fingers of the robot-manipulator. For example we discussed a two legged humanoid robot with five fingers pair arms or a monkey-robot with twenty arms and legs fingers. The robot can hold the object by one and grasp by two or three fingers. An object grasping problem is equivalent to the problem of the walking robot with  $n$  legs. Consider a grasp with  $m$  fingers.

The work [1] is devoted to control of the manipulator along communication. The difference our work form another authors speaking about grips is that each finger contacts an object in one foothold. And we speak

more not about control, but about static stability during transferring the object by a grip.

There is an analogy of this problem to the problem of walking robot dynamics on one-side constraint. While the general walking robot motion on a plane was analyzed in detail in [2] the case of the dynamics on a curved surface is far more complicated. Model dynamics and control problems was considered in [3]. Equilibrium conditions for a solid on a rough plane was considered in [4]. Walking robot parameters optimization for the motion in tubes was considered in [5]. The special case of a robot with eight legs whose up porting points are restricted to the inner surface of a tube was considered in [6]. In the present work, we consider the more general case of a robot with three arbitrary supporting points on a rough cylinder and on a curved surface.

Let the point  $O$  is an origin fixed in absolute space. Suppose that robot arms fingers accomplish the desired motion with respect to the body of the robot.

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Using general dynamics theorems to describe the cylinder motion, we obtain six different equations for the cylinder dynamics from the momentum and angular momentum theorems [7], [8]. Among them there are three equations of the body translation with point  $A$  and another three describe body rotation about point  $A$ . For prescribed motion be realized then reaction in  $m$  footholds should satisfy following kinetostatic equations [9], [10]:

$$\sum_{i=1}^m \tilde{\mathbf{R}}_i = -\tilde{\mathbf{\Phi}}, \quad \sum_{i=1}^m \tilde{\mathbf{r}}_i \times \tilde{\mathbf{R}}_i = -\tilde{\mathbf{M}}, \quad (1)$$

where  $\tilde{\mathbf{R}}_i$  is reaction component,  $\tilde{\mathbf{r}}_i$  corresponds to the  $i$ -th finger supporting point vector,  $\tilde{\mathbf{\Phi}}$  is the sum of the external active forces plus time derivative of desired momentum, and  $\tilde{\mathbf{M}}$  is the sum of external active forces momentum and time derivative of desired angular momentum with respect to the point  $O$ . In two vector equations in (1), the former corresponds to the momentum of the object (and is equivalent to three scalar equations when projected onto the basis vectors), while the latter defines the desired change of the angular momentum.

Assuming that  $\tilde{\mathbf{\Phi}}$  is orthogonal to  $\tilde{\mathbf{M}}$ , we obtain [11] that the system  $\{\tilde{\mathbf{\Phi}}, \tilde{\mathbf{M}}\}$  can be also used at the point  $C$

$$\tilde{\mathbf{r}}_C \times \tilde{\mathbf{\Phi}} = \tilde{\mathbf{M}}, \quad \tilde{\mathbf{r}}_C = -\frac{\tilde{\mathbf{M}} \times \tilde{\mathbf{\Phi}}}{\tilde{\Phi}^2}, \quad \tilde{\Phi} = |\tilde{\mathbf{\Phi}}|,$$

where  $\tilde{\mathbf{r}}_C$  is the vector  $OC$ , and  $C$  corresponds to the point at which the resultant of the reactions is acting.

Further problem of reactions distribution  $\tilde{\mathbf{R}}_i$  in some fixed point of time is investigated by the proposal that force  $\tilde{\mathbf{\Phi}}$  is acting at the point  $\tilde{\mathbf{r}}_C$  and force moment there is zero. Motion equations (1) for finding reactions of fingers prescribed motion can be transformed [12]:

$$\sum_{i=1}^m \tilde{\mathbf{R}}_i = \tilde{\mathbf{\Phi}}, \quad \sum_{i=1}^m \tilde{\mathbf{r}}_i \times \tilde{\mathbf{R}}_i = \tilde{\mathbf{r}}_C \times \tilde{\mathbf{\Phi}}. \quad (2)$$

For example point  $C$  can be the grasping object center of mass.

Assuming that the robot footholds are on the surface of a rough cylinder of radius  $\rho$  with a friction coefficient  $k$ , we introduce the coordinate system  $Oxyz$  such that the axis  $Ox$  is directed along the cylinder axis (so that the projection of  $\tilde{\mathbf{\Phi}}$  on the axis  $Ox$  is negative – see Fig. 1.), the axis  $Oz$  is parallel to the vector  $\tilde{\mathbf{\Phi}}$ , and the angle between the cylinder axis and the vector  $\tilde{\mathbf{\Phi}}$  is  $\alpha$ .

The problem of finding the reaction forces (2) is similar to the foothold reactions distribution problem for walking robot, when the footholds are on the external surface of a rough inclined cylinder where the axis has an angle  $\alpha$  with respect to the vector  $\tilde{\mathbf{\Phi}}$ . It

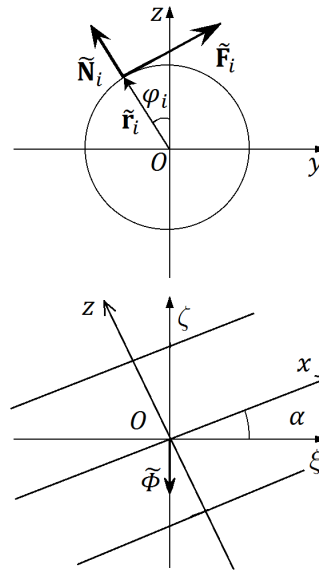


Figure 1. Cylinder

has been considered in [10] the problem of searching of the reactions components along the cylinder axis when  $\alpha = 0$ .

In the coordinates  $Oxyz$  we define  $\tilde{\mathbf{R}}_i = (\tilde{R}_i^x, \tilde{R}_i^y, \tilde{R}_i^z)$ ,  $\tilde{\mathbf{r}}_C = (\tilde{x}_C, \tilde{y}_C, \tilde{z}_C)$ , and  $\tilde{\mathbf{\Phi}} = (-\tilde{\Phi} \sin \alpha, 0, -\tilde{\Phi} \cos \alpha)$ ,  $i = 1, \dots, m$ . In case of a one-sided surface, and the grasp inside the cylinder, we have additional restrictions on normal reactions  $\tilde{N}_i$  [14]:

$$\tilde{N}_i = \tilde{\mathbf{R}}_i \cdot \mathbf{e}_v^i \geq 0, \quad (3)$$

where  $\mathbf{e}_v^i$  is an external normal to  $i$ -th supporting point on the cylinder, while the tangential components are given by  $\tilde{\mathbf{F}}_i = \tilde{\mathbf{R}}_i - \tilde{N}_i \mathbf{e}_v^i$ .

For the reactions to be in the friction cones (2), we have following inequalities:

$$|\tilde{\mathbf{F}}_i| \leq k \tilde{N}_i, \quad (4)$$

i.e. the tangential reactions  $\tilde{\mathbf{F}}_i$  are restricted by Coulomb limiting friction value. When  $\tilde{\mathbf{F}}_i$  exceeds this limiting value, the robot legs and arms begin to slide along a surface.

The reaction distribution problem then reduces to the solution of equations (2), and inequalities (3), (4), for reactions limited to the friction cones. The restricted motion can only be realized if the solution of Eqns. (2)-(4) does exist.

The same inequalities are for walking robot on the cylinder [10]. If the grasp is out the cylinder this inequalities (3) have opposite sign.

For example if  $m$  is even. And one of each par of the supporting points is on and another is in the thin

surface such that we consider them like one geometrical point. Then we need only inequalities (4).

For  $\mathbf{r}_i = \tilde{\mathbf{r}}_i/\rho = (x_i, y_i, z_i)$ , in the coordinate:  $\mathbf{r}_i = (x_i, -\sin \varphi_i, \cos \varphi_i)$ ,  $\mathbf{e}_i^y = (0, -\sin \varphi_i, \cos \varphi_i)$ ,  $N_i = \tilde{\mathbf{N}}_i/\tilde{\Phi} = (0, -N_i \sin \varphi_i, N_i \cos \varphi_i)$ , where  $\varphi_i$  is the angles between axis  $Oz$  and cylinder normal  $\mathbf{e}_i^y$ . We define  $\mathbf{e}_x$  as the unitary vector in the  $Ox$  axis, while  $\mathbf{e}_i^r = (0, \cos \varphi_i, \sin \varphi_i)$  as the tangential to the cylinder. Then the tangential reaction:  $\mathbf{F}_i = (F_i^x, F_i^{yz} \cos \varphi_i, F_i^{yz} \sin \varphi_i)$ , where  $F_i^x = \mathbf{F}_i \cdot \mathbf{e}_x$ ,  $F_i^{yz} = \mathbf{F}_i \cdot \mathbf{e}_i^r$ ,  $\mathbf{R}_i = \tilde{\mathbf{R}}_i/\tilde{\Phi} = (R_i^x, R_i^y, R_i^z)$ ,  $\mathbf{r}_C = \tilde{\mathbf{r}}_C/\rho = (x_C, y_C, z_C)$ .

Let  $p = R_1^x - R_2^x$ . We further define the coordinate differences, and the supporting points difference of angles of axis  $Oz$  are  $\Delta x = x_2 - x_1$ ,  $\Delta y = y_2 - y_1$ ,  $\Delta z = z_2 - z_1$ ,  $\Delta \varphi = \varphi_2 - \varphi_1$ , and  $s_{21} = \sin \varphi_2 - \sin \varphi_1$ ,  $c_{21} = \cos \varphi_2 - \cos \varphi_1$ . We then project system (2) onto the axes  $Oxyz$ . For arbitrary surface we find that the second equation of (2) (corresponding to the moment) has the skew-symmetric matrix with respect to the component  $R_i^x$  [10]. These are 2 independent equation, while the third equation corresponds to the restriction of the point  $C$  to the plane containing the two footholds. As a result, the system (2) yields 5 independent equation and a restriction.

## 2. A Three-finger Grasp

During the robot motion one, two and three supporting points phases are changed. For example, australian lizards - yellow-bellied three-toed skinks (*saiphos equalis*). First, we consider the one-supporting phase of the grasp. Let  $m = 1$ , then the motion existing condition is reaction is equal to force  $\Phi$  and supporting point and the point  $C$  are on the line along  $\Phi$ , while the angle between  $\Phi$  and the normal do not exceed the friction angle.

If the grasp inside the surface then point  $C$  is under the surface. In opposite case the grasp is under the surface. Then point  $C$  is inside the surface. Or if one finger out the cylinder, the center mass of an object is up the finger. And the angle between the weight and the normal not exceed friction angle.

If  $m$  is even. And one of each par of the supporting points is on and another is in the thin surface such that we consider them like one geometrical point. Then it does not matter where the point  $C$  is on the line.

Let  $n = 2$ , and  $x_1 \neq x_2$ . Then  $p = F_2^x - F_1^x$ , and

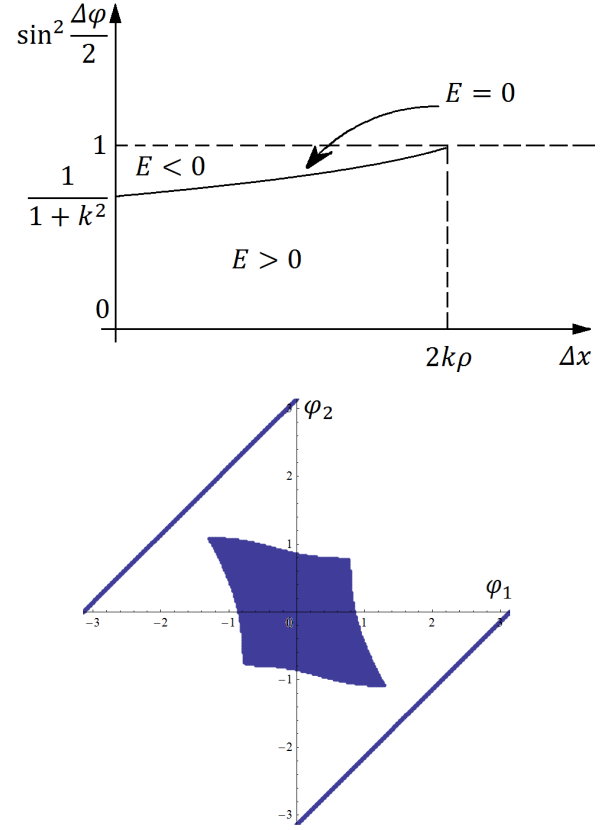


Figure 2. The analytical and the numerical parameter diagrams

from (2):

$$\begin{aligned} F_1^x &= (\sin \alpha + p)/2, & F_2^x &= (\sin \alpha - p)/2, \\ N_1 &= \frac{-p \sin^2 \frac{\Delta \varphi}{2} + (x_2 - x_C) \cos \varphi_1 \cos \alpha}{\Delta x} + N_1^\alpha, \\ N_2 &= \frac{-p \sin^2 \frac{\Delta \varphi}{2} + (x_C - x_1) \cos \varphi_2 \cos \alpha}{\Delta x} + N_2^\alpha, \\ F_1^{yz} &= \frac{-p \sin \Delta \varphi + 2(x_2 - x_C) \sin \varphi_1 \cos \alpha}{2\Delta x} + F_1^{(yz)\alpha}, \\ F_2^{yz} &= \frac{p \sin \Delta \varphi + 2(x_C - x_1) \sin \varphi_2 \cos \alpha}{2\Delta x} + F_2^{(yz)\alpha}, \\ \tan \alpha &= \frac{\Delta x (\sin \varphi_2 + y_C) + (x_C - x_2) s_{21}}{y_C c_{21} + z_C s_{21} - \sin \Delta \varphi}, \end{aligned} \quad (5)$$

where  $N_i^\alpha$  and  $F_i^{yz}$  are the functions of  $x_i$ ,  $\varphi_i$ ,  $y_C$  and  $z_C$ . The conditions (4) can be displayed in the form

$$E p^2 + B_1 p + C_1 \leq 0, \quad E p^2 + B_2 p + C_2 \leq 0, \quad (6)$$

where

$$E = (\Delta x)^2 + \sin^2 \Delta \varphi - 4k^2 \sin^4 \left( \frac{\Delta \varphi}{2} \right),$$

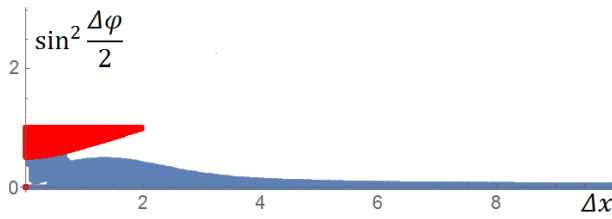


Figure 3. For  $\alpha = \pi/4; x_2 = -x_1, \varphi_2 = -\varphi_1$

$B_i, C_i$  are the functions of  $x_i, \varphi_i, x_C, y_C$  and  $z_C$ .

The boundaries between different regimes can be determined analytically. For example, in the case of  $E < 0$ , the solution exists, and can be obtained analytically [10], as shown in Fig. 2, on the left. Note that in this case it's limited to the range  $\Delta x \leq 2k\rho$ . In contrast to this behavior, for  $E \geq 0$  there is no such restriction and an additional step is required to address the question of the existence of the solution. At the point  $(0, 0)$  we find  $E = 0$ , which means that two footholds are orthogonal to the cylinder axis. Here, two possible solution are either identical, or limited to a single diameter. In the latter case, point C and the reaction have to be in one plane, parallel to force  $\Phi$ , and the problem has a solution.

For the desired legs or fingers configurations and given point C, the problem can be solved numerically. In Fig. 2, on the right, we present the numerical solution for the example when  $x_2 = -x_1 = \rho = k = 1$ . Note that in this case  $E > 0$ .

Specifically, the condition (6) was analyzed in two cases, when  $E = 0$  and  $E > 0$ , and when the solution of the problem does exist, the solutions were shown in the plot.

For  $E > 0$ , we need to consider two conditions. First is the restriction on the determinants  $D \geq 0$ , while the second is the requirement of a non-empty intersection of the set of points of the intervals between the roots of quadratic equations. From this plot we see that, if two points are on one diameter, then the solution of the reaction distribution problem exists. The two lines in the plot, correspond to  $\varphi_1 = \varphi_2 + \pi$  or  $\varphi_1 = \varphi_2 - \pi$ . The rhombus form represents the requirement on the determinants  $D_i \geq 0$ , while additional conditions further restrict the range [12].

In Fig. 3 we present the results for  $E > 0$  and  $E \leq 0$ , when  $x_2 = -x_1, \varphi_2 = -\varphi_1$  and shows the case of  $\alpha = \pi/4$ . The figures for  $\alpha = 0$  and increased to  $\pi/2$  are shown in [13]. Note that when  $\alpha = \pi/2$ , the solutions exists only for diametrical footholds.

For two-finger robot when  $E$  is negative, the solution exists, and obtained analytically [14]. Using numerical simulations we explain the reaction distribution problem existing and build this problem so-

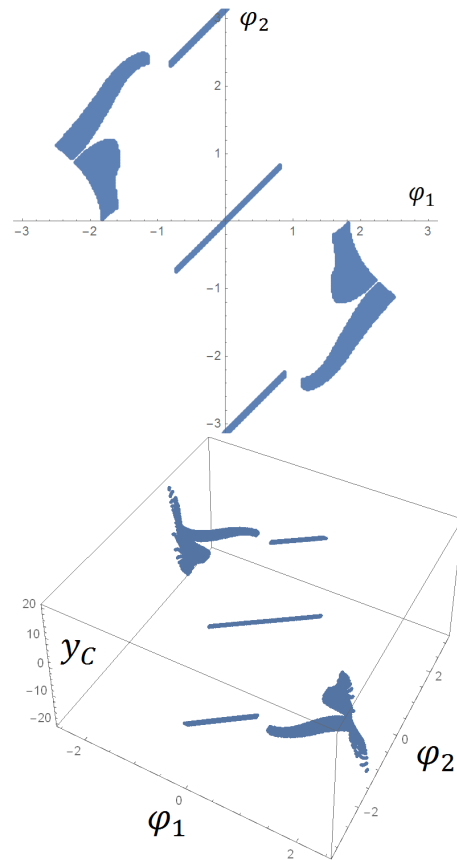


Figure 4. Admissible area for  $\alpha = \pi/3; \Delta x = 1, 1$ .

lution existing fields for given footholds and point C position [15]. For example, for two-foothold phase, we consider symmetric, about point C, along and orthogonal cylinder axis, robot configurations [16]. For first of these configurations examined cases with non-negative  $E$  coefficient, for distances between point C and footholds [17]. Reactions distribution problem solution existing fields constructed on the two angles plane, correspond to footholds projections on the cylinder base and three dimensional fields which supplement this plane by point C z-coordinate altitude [18]. When  $x$  equals to 1,1 for  $\alpha$  equals  $\pi/3$  in three-dimensional fields observed bundles of separate points, Fig. 4. That means that the point C altitude position more harsh change while changing the angles.

A possible three fingers configuration: one of the fingers is in vertical plane containing cylinder axis and another two are in the plane orthogonal to the axis.

### 3. Conclusion

A solution of the problem of reaction distribution with respect to supporting points on the cylinder, both for walking robot and its grasp, demonstrates a

great variety of cases that must be analyzed individually. Therewith, it turns out that there exist isolated equilibriums, even if support polygon does not degenerate. For example, for a smooth cylinder or surface. Such a specify principally differentiates the problem of reaction distribution on a cylinder or a surface from the problem of reaction distribution on a horizontal plane, two or some planes. For a set of equilibriums to be connected in the case of three supporting points, this provides a means for comfortable quasi stationary motion, it is necessary to select supporting points in a special way, so that one of them be located in the lower or upper element of the smooth cylinder. For rough cylinder this solution exists. If between the surface of the cylinder and legs of walking robot or it fingers, viscous friction arises, then this condition is essential, therewith, given dry friction, it is of lesser importance for algorithm of motion control.

The motion of the walking robot between two horizontal supporting planes has additional possibilities for the distributions of reactions in comparison with the motion on one supporting plane for the cases where a projection of the center of mass of the robot, or part of planes, is beyond the supporting polygon. The proposed method of distribution of reaction allows one to estimate the limitations of these possibilities and to use them profitably.

During the robot motion, one, two and three supporting point phases are changed. And for example the humanoid robot with five arm fingers can hold the object by one and grasp by two or three-fingers. The reaction distribution problem have a solution in following cases. Let we give some examples.

1. One-supporting point phase. So the motion existing condition is reaction is equal to force  $\Phi$  and supporting point and the point  $C$  are on the line along  $\Phi$ . And the angle between  $\Phi$  and the normal not exceed friction angle.
  - 1.1 If the grasp inside the surface then point  $C$  is under the surface. In opposite case the grasp is under the surface. Then point  $C$  is inside the surface.
  - 1.2 If  $m$  is even. And one of each par of the supporting points is on and another is in the thin surface such that we consider them like one geometrical point. Then it does not matter where the point  $C$  is on the line.
2. Two-supporting point phases. In case when the grasp is inside the cylinder. The point  $C$  and the reactions have to be in the plane parallel to force  $\Phi$ .
  - 2.1 If supporting points are on one diameter.

- 2.2 When coefficient  $E < 0$ . And in some fields with connected set of points, when  $E \geq 0$ . Robot can hold the cylinder by two fingers on one diameter.

3. A robot can hold the horizontal cylinder by three fingers. Let one of the points is in vertical plane containing cylinder axis and another are in the plane orthogonal to the axis. Without friction, the cylinder center of mass has to be in the vertical plane that contain the cylinder axis. The supporting points are on the external surface of the lower semi-cylinder and the center mass of the cylinder is in the footholds triangle. If the first supporting point is in the lower semi-cylinder and two another are on the upper, the center of mass has to be out of the footholds triangle.
4. If  $m$  is even. And one of each par of the supporting points is on and another is in the thin surface such that we consider them like one geometrical point. Then it does not matter where is the point  $C$ .

So the robot can transfer the cylinder by one, two or three fingers.

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