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Broadband liner impedance eduction for multimodal acoustic propagation in the presence of a mean flow[1]

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Modeling of acoustic propagation in a duct with absorbing treatment is considered. The surface impedance of the treatment is sought in the form of a rational fraction. The numerical model is based on a resolution of the linearized Euler equations by finite difference time domain for the calculation of the acoustic propagation under a grazing flow. Sensitivity analysis of the considered numerical model is performed. The uncertainty of the physical parameters is taken into account to determine the most influential input parameters. The robustness of the solution vis-a-vis changes of the flow characteristics and the propagation medium is studied.

Keywords: sensitivity, grazing flow, impedance, Sobol's method

1. Introduction

Acoustic liners mounted in the walls of aircraft engine nacelles are commonly used to achieve noise reduction. Due to the increase of the engine size to obtain ultra-high by-pass ratio, the size of nacelles is expected to be shortened in the next generation of aircrafts and consequently, the efficiency of current liners will decrease.

The key parameter to evaluate noise reduction of novel concepts is the surface impedance of the liners. To determine it in situ, inverse techniques based on propagation models for the lined duct are becoming popular because of their convenience and advantages [1–4]. These methods are performed through the measurement of the acoustic characteristics at selected locations outside the liner and have an extra advantage of not destroying the liner sample. In this contribution a recently developed by Troian et al. [5] broadband impedance eduction method is considered. It identifies the surface impedance of acoustic liners, mounted in the walls of aircraft engine nacelles, from measurements on a test rig. A numerical model of an acoustic liner under a grazing flow is undertaken by considering finite-difference time-domain simulations and the Euler equations for the acoustic propagation. A broadband impedance model is used to prescribe time-domain boundary conditions. The identified impedance could be influenced by many sources of error either from the numerical simulation, physical assumptions or experimental errors. The quality of the impedance eduction process is critically dependent on the uncertainties of the input parameters of the model. However the uncertainty analysis has rarely been considered. Several works can be mentioned. Brown et al. in [6], identified the measurement uncertainty of the impedance eduction process. The studied parameters are Mach number, static temperature and pressure. Schultz et al. considered the impedance measurement by two-microphone method in [7]. The multivariate uncertainty analysis technique and the Monte

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Carlo methods for estimating the experimental uncertainties were presented. Zhou and Boden in [8] presented a systematic multivariate uncertainty analysis technique for estimating experimental uncertainties of educed impedance. Different levels of component uncertainties have been studied for their contribution to the overall impedance results.

We propose here to conduct a sensitivity analysis of a numerical model of acoustic propagation in the treated duct under the grazing flow. Three physical parameters of the experimental devices, namely the temperature, the flow velocity and the acoustic treatment length are studied. Sensitivity indices are calculated following the Sobol methodology.

2. Multimodal acoustic propagation modelling

A rectangular duct including a treated section and the presence of a mean flow is chosen as the test channel. A sketch is given in fig. 1. The source and the exit planes of the computational domain are located at $x = 0$ and $x = L_x$, respectively. The dimensions of the duct cross-section are $L_y \times L_z$. The lower and two-side walls are rigid. The upper wall is also rigid except of the lined region ($L_1 < x < L_2$ in fig. 1). The dimensions of the channel are $L_x = 0.812$ m, $L_1 = 0.203$ m, $L_2 = 0.609$ m, $L_y = L_z = 0.051$ m. Sound propagation in a lined duct is modeled by the Euler equations, linearized around a given mean flow of density ρ_0 and velocity $\mathbf{V_0}$. The mean pressure is assumed to be constant, the longitudinal pressure gradient is small and the mean flow is homentropic. The acoustic velocity v and the acoustic pressure p are obtained by solving these equations written for an ideal gas :

$$\frac{\partial p}{\partial t} + (\mathbf{V_0} \cdot \nabla)\, p + \rho_0 {c_0}^2 \nabla \cdot \mathbf{v} = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{V_0} \cdot \nabla)\, \mathbf{v} + (\mathbf{v} \cdot \nabla)\, \mathbf{V_0} + \frac{1}{\rho_0} \nabla p = 0$$

where t is the time and c_0 is the celerity of sound in the air.

These equations are discretized by low-dispersion and low-dissipation explicit numerical schemes, developed in computational aeroacoustics [9,10]. Optimized finite-difference schemes and selective filters over 11 points are used for spatial derivation and grid-to-grid oscillations removal, respectively.

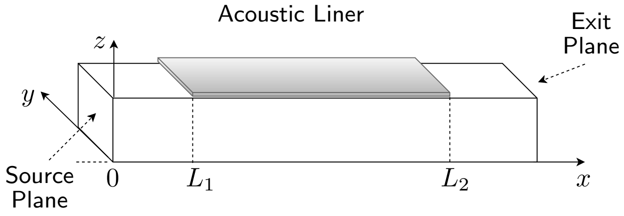

Fig. 1. Sketch of the grazing flow impedance tube

The centered fourth-order finite-difference scheme of Bogey and Bailly [10] and the centered sixth order selective filter of Bogey et al. [11] are applied for the interior points, separated by at least five points from the boundary. For the boundary points in each direction, the eleven-point non-centered finite-difference schemes and selective filters of Berland et al. [9] are implemented. The optimized six-stage Runge–Kutta algorithm proposed by Bogey and Bailly [10] is used for time integration. The time-domain impedance boundary condition is presented in the next subsection. The inlet and outlet sections are assumed to be anechoic. Damping zones, including the non-reflecting boundary conditions of Bogey and Bailly [12] are thus implemented.

The absorbing treatment is modeled with broadband impedance condition, developed in the time domain, to avoid the prescribing the surface impedance for each frequency of interest. The multipole impedance model :

$$\begin{aligned} Z(\omega) = Z_\infty + \sum_{k=1}^{P} \frac{A_k}{\lambda_k - i\omega} + \\ \sum_{k=1}^{S} \left(\frac{B_k + iC_k}{\alpha_k + i\beta_k - i\omega} + \frac{B_k - iC_k}{\alpha_k - i\beta_k - i\omega} \right) \end{aligned} \qquad (1)$$

is introduced. Choosing Z_∞, A_k, B_k, C_k and λ_k, α_k, β_k real positive ensure that the impedance model is causal and real. The passivity condition has to be checked for each set of the coefficients to guarantee the impedance being physically admissible [13]. The formulation (3) brings two advantages. First, a broadband impedance model is straightforwardly obtained. Second, when using time-domain methods, the impedance condition formulation leads to a convolution, which is computationally expensive. Using the multipole impedance model, the convolution can be evaluated at a small computational cost [14]. In the following, during the impedance eduction procedure, the coefficients of the acoustic impedance are determined by minimizing the error function between the calculated and the measured transmission loss (TL). Pressure for frequencies below the duct cut-off frequency is used additionally in order to improve the robustness of the results. Details can be found in Troian et all [5].

3. Sensitivity analysis of the numerical model

The sensitivity analysis has two main purposes. The first is to identify the input variables that have a strong influence on the output of the model. These variables have to be determined precisely to improve the accuracy of the model. The second aim is to identify, on the contrary, the one or more input variables that have less influence on the output. It is then not necessary to have a major precision for these variables. Thus, for models with a large number of inputs variables, the sensitivity analysis allows determining variables that have a significant impact on output and simplifies the model by neglecting the precision for input variables with small influence. The method of the impedance identification considered here consists in two problems. The first is the direct problem of sound propagation in a duct with acoustic treatment under a grazing flow. The second corresponds to the inverse problem of identifying the impedance parameters basing on comparison between measurements and simulated results. Present paper is devoted to the sensitivity analysis of the direct problem.

3.1. Parameters that are considered in the sensitivity study

The transmission loss is usually used to characterize the properties of an absorbent treatment. It represents the reduction in acoustic power between the entrance and exit of the duct due to the presence of absorbing treatment. Among the physical parameters of the measurements facility, Mach number, sound speed, sound density and liner geometry are the parameters that have to be measured precisely and that can influence much on the resulting transmission loss. These parameters are calculated as following: $M = V/c_0$, where M is the Mach number; V is the local flow velocity and c_0 is the speed of sound in the medium. For the perfect gas, celerity of the sound and density are given by $c_0 = \sqrt{\gamma r T}$ and $\rho_0 = p_0/(rT)$, where γ and r are specific heat ratio and specific gas constant for dry air respectively. Thus, the principal physical parameters can be defined through two independent characteristic, they are the local flow velocity V and temperature T. These two variables together with the length of the liner are the parameters for which sensitivity analysis will be conducted.

3.2. Sobol method of sensitivity analysis

Sensitivity analysis is a study of how the variation in the output of a model (numerical or otherwise) depends, qualitatively or quantitatively, on the model inputs. The concept of using variance as an indicator of the importance of an input parameter is the basis for many variance-based sensitivity analysis methods. To perform global sensitivity analysis the following steps are needed:

- Model input parameters are selected and probabilities and distributions are assigned.
- A set of random input vectors is generated from the associated probability distribution for each parameter.
- The model is evaluated for each set of input vectors.
- The sensitivity indices, which are the fractional contribution to the output variance due to uncertainties in the inputs, are calculated.

The input parameters are ranked according to their influence on the output. While there are many methods available for analyzing the decomposition of variance as a sensitivity measure, the Method of Sobol [15] is one of the most established and widely used methods and is capable of computing the 'Total Sensitivity Indicies' (TSI), which measures the main effects of a given parameter and all the interactions (of any order) involving that parameter. Sobol's method uses the decomposition of variance to calculate the Sobol's sensitivity indexes. The basis of the method is the decomposition of the model output function $y = f(x)$ into summands of variance using combinations of input parameters in increasing dimensionality. To determine the sensitivity of the output to the variation of an input parameter, an input factor space,$\Omega^n = (x|0 \leqslant x_i \leqslant 1;\ i = 1, ...,\ n)$ is defined, where n is the number of variables. For a given model f linking input parameters $x = (x_1, ...x_n)$ to a scalar output $y = f(x)$, there exists a unique partition of f so that

$$y = f(x_1, x_2, ...x_n) = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{i \leqslant j} f_{ij}(x_i, x_j) + ... + f_{1...n}(x_1, ..., x_n)$$

where f_0 is the mean of f, provided that each function f_I for a given set of indices $I = i_1, ..., i_n$, involved in the decomposition has zero mean over its range of variation :

$$\int_0^1 f_I(x_I) dx_I = 0$$

The total variance D of $f(x)$ is defined to be

$$D = \int_{\Omega^k} \left(f^2(x) - f_0^2\right) dx$$

The partial variance is therefore the variance of f_I

$$D_I = \int_0^1 f_I^2(x_I)dx_I$$

and the sensitivity index relative to the set I is expressed as the ratio of the variance of the function f_I to the total variance of the model: :

$$S_I = \frac{D_I}{D} \tag{2}$$

Another important sensitivity measure for a given parameter i is the total sensitivity index $S_{I_{\mathrm{tot}}}$, defined as the sum of the indices of all sets of parameters I to which i belong

$$S_{I_{\mathrm{tot}}} = 1 - \frac{D_{\sim I}}{D}$$

where $D_{\sim I}$ is the partial variance of all the parameters except of I. The first-order index represents the share of the output variance that is explained by the considered parameter alone. Most important parameters therefore have high index, but a low one does not mean the parameter has no influence, as it can be involved in interactions. The total index is a measure of the share of the variance that is removed from the total variance when the considered parameter is fixed to its reference value. Therefore parameters with low $S_{I_{\mathrm{tot}}}$, can be considered as non-influential.

In practice, Sobol's method is relatively easy to implement using Monte Carlo based integration. Sobol's first order and total effect sensitivity indices can be implemented by expressing equation (2) in a discrete form following the procedure described in [16]. First, two matrices of data has to be generated, $A = [a_{ij}]$ and $B = [b_{ij}]$, $i = 1, ..n$, $j = 1, ..N$. After this a matrix C_i has to be formed by all columns of B except the ith column, which is taken from A. The model output for all the input values in the sample matrices A, B, and C_i is computed to obtain three vectors of model outputs $y_A = f(A)$, $y_B = f(B)$, $y_C = f(C_i)$. With :

$$f_0^2 = \left(\frac{1}{N} \sum_{j=1}^{N} y_A^{(j)} \right)^2 ,$$

first-order sensitivity indices are estimated as follows :

$$S_i = \frac{y_A \dot{y}_{C_i} - f_0^2}{y_A \dot{y}_A - f_0^2} = \frac{\left(\frac{1}{N} \sum_{j=1}^{N} y_A^{(j)} y_{C_i}^{(j)} - f_0^2 \right)}{\left(\frac{1}{N} \sum_{j=1}^{N} y_A^{(j)} y_A^{(j)} - f_0^2 \right)}$$

Similarly, the method total-effect indices are calculated as follows:

$$S_{I_{\mathrm{tot}}} = 1 - \frac{y_B \dot{y}_{C_i} - f_0^2}{y_A \dot{y}_A - f_0^2} = 1 - \frac{\left(\frac{1}{N} \sum_{j=1}^{N} y_B^{(j)} y_{C_i}^{(j)} - f_0^2 \right)}{\left(\frac{1}{N} \sum_{j=1}^{N} y_A^{(j)} y_A^{(j)} - f_0^2 \right)}$$

4. Results

Following the algorithm presented in the previous section, numerical results for sensitivity analysis of acoustic propagation under the grazing flow in the duct with acoustic treatment are obtained. The input parameters of the system that are considered for a sensitivity analysis are given in the Table 1.

Parameters are supposed to be independent, distributed as random variables using Latin hypercube sampling (LHS) [17]. The sample size is $N = 4000$. The liner impedance correspond to the impedance of a ceramic tubular liner studied in [3] for $M = 0$. First order effects and total effects of the TL due to changes in measurements characteristics were calculated. Numerical values are presented in the Fig. 2 and 3.

Comparing the first order of sensitivity and full sensitivity, it can be concluded that the studied parameters are almost independent. For all the frequencies the most important parameter on the value of TL is the flow velocity, and the least influential parameter is the temperature. This implies that the Mach number, which depends on the flow velocity, must be known precisely in order to limit the uncertainty about the value of TL. It is also important to take into account the uncertainty of the number of Mach. Conversely, it is not necessary to have an important precision on temperature and, therefore, the speed of sound and the density of air. For a frequency $f = 1100$ Hz, the index of sensitivity associated to the length of treatment becomes important. Indeed, this frequency corresponds to the resonance frequency of treatment. Therefore, the TL will vary strongly with small variations in the length of treatment, which involves a high sensitivity of TL vis-a-vis the size of the treatment.

Table 1. Numerical values of parameters used in the simulation

Input parameters	Val. min	Val. max
Temperature, T, C	0	35
Mean flow, V, m/s	0	170
Liner length, $L_2 - L_1$, m	0.6	0.7

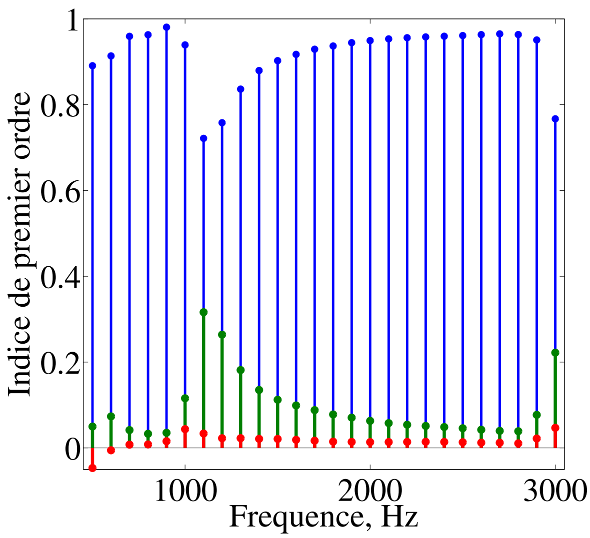

Fig. 2. First order indices of temperature T (red), mean flow velocity V (blue) and liner length (green) using Sobol's method

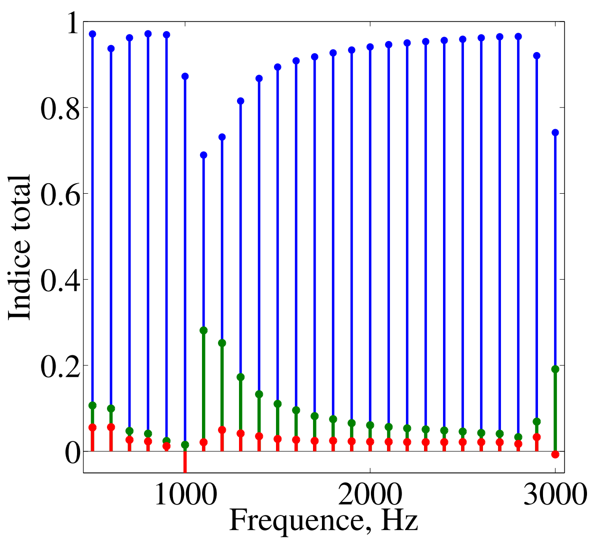

Fig. 3. Total indices of temperature T (red), mean flow velocity V (blue) and liner length (green) using Sobol's method

5. Conclusion

A sensitivity analysis for the numerical model of an acoustic propagation in a duct under a grazing flow is performed. Sobol's method for global sensitivity analysis was presented and the total sensitivity and first-rate indices were calculated. The input parameters were classified according to their contribution to the overall variance of the function. The obtained results allow identifying the physical parameters that with sufficient precision can reduce uncertainty for measured transmission loss values. This quantity is particularly used in the inverse identification methods of impedance acoustic treatments. By limiting the uncertainty on the values of transmission loss, it is expected that the uncertainty in the identified impedance also decreases.

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