УДК 532.546

MATHEMATICAL MODEL AND SOME NONLINEAR EFFECTS OF HEAT AND MASS TRANSFER IN MULTIPHASE MEDIA UNDER ACTION OF HIGH-FREQUENCY ELECTROMAGNETIC RADIATION[1]

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Abstract. The application of high-frequency electromagnetic radiation is one of the promising methods in oil and gas technologies, that can be well use for struggle against pitch, paraffin or gas-hydrate fall-out in wells and in pipelines, for action on reservoir and for other purposes. During in the operation of wells on their surfaces the asphalt-pitch-paraffin fall-out are occurs caused of lowing of temperature and pressure. These fall-out fills the space between pipes, that bring to a stop of well sometimes. The high-frequency electromagnetic heating is convenient mean (and the only possible way in some cases) to eliminate these complications. By that the experiments and the numerical simulation shows, that some nonlinear effects are possible. These effects are caused due to some media have temperature dependence of absorption coefficient. This dependence can be have more or less expressed"resonance" view, to say have maximum by certain temperature. The position, height and width of that "resonance" dependences of electromagnetic radiation frequency and of dielectric properties of medium. These nonlinear properties can be used to rise of heating efficiency. If the radiation frequency is correct, one can to realize the heating in a "temperature wave" regime, that essentially rises the velocity of heating. And what is more, if one will use the nonlinear dependences $\alpha(T)$, one can to realize a "reverse temperature wave", that will move backwards: from far end of tube to the source of radiation, i.e. one can receive effects, those are impossible by usual conditions. In this paper the temperature dependence of absorption coefficient is investigated and the mathematical modelling of high-frequency electromagnetic heating (with due regard for this dependence) is carried out.

[1] Работа была доложена на Международной конференции по многофазным системам, посвященной 60-летию академика Р. И. Нигматулина (15–17 июня 2000 года, г. Уфа)

Keywords: mathematical model, nonlinear effects, multiphase media, electromagnetic radiation

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1 The temperature dependence of absorption coefficient of electromagnetic radiation

The absorption coefficient of electromagnetic radiation a in medium by $\varepsilon''<<\varepsilon'$ is:

$$\alpha = \frac{\omega\varepsilon''}{C_R\sqrt{\varepsilon'}}, \tag{1}$$

where ε', ε'' — are real and imaginary parts of dielectric permittivity; C_R — velocity of light in vacuum; $\omega = 2pf$.

In terms of semiempirical Cole-Cole model of dielectric relaxation the real and imaginary parts of dielectric permittivity are:

$$\begin{gathered}\varepsilon' = \varepsilon_\infty + \\ + \frac{(\varepsilon_s - \varepsilon_\infty)\left[1 + (\omega\tau_0)^{1-\beta}\sin(\beta\pi/2)\right]}{1 + 2(\omega\tau_0)^{1-\beta}\sin(\beta\pi/2) + (\omega\tau_0)^{2(1-\beta)}},\end{gathered} \tag{2}$$

$$\varepsilon'' = \frac{(\varepsilon_s - \varepsilon_\infty)[1 + (\omega\tau_0)^{1-\beta}\cos(\beta\pi/2)]}{1 + 2(\omega\tau_0)^{1-\beta}\sin(\beta\pi/2) + (\omega\tau_0)^{2(1-\beta)}}, \tag{3}$$

where ε_S, ε_∞ are static and high-frequency limits ε' correspondingly, τ_0 — the most probable relaxation time of dielectric molecules; β — parameter, that determines the width of relaxation time spectrum, its values are in limits $0\leq\beta<1$.

If all of dielectric molecules have the sole relaxation time τ_0, than $\beta = 0$, Cole-Cole model reduces to the Debye model, and equations (2), (3) become the following view:

$$\varepsilon' = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + (\omega\tau_0)^2}, \tag{4}$$

$$\varepsilon'' = \frac{\varepsilon_s - \varepsilon_\infty}{1 + (\omega\tau_0)^2}\,\omega\,\tau_0. \tag{5}$$

Parameters ε_S, ε_∞, β have a weak temperature dependence [1, 2], therefore one can to take in the first approximation, that the dependence $\alpha(T)$ is determined only by the temperature dependence of relaxation time. This dependence is given as a rule in the following view:

$$\tau_0 = \tau_\infty \exp(E_\tau/RT) = \tau_\infty \exp(T_\tau/T), \tag{6}$$

where E_τ, $T_\tau = E_\tau/R$ are activation energy and activation temperature, τ_∞ — is the high-temperature limit of relaxation time, R is the gas constant per mole.

If $\omega\tau_0$ changes from 0 to ∞, the real part of dielectric permittivity changes from ε_S to ε_∞. These parameters differ weakly, therefore substitution of value $\sqrt{\varepsilon'}$ on its average $\sqrt{(\varepsilon_S+\varepsilon_\infty)/2}$ in the equation (1) led to mistake for a not larger, that a few per centum. For example, the mistake for the Russkoe oil will about 7%; that is not larger, than the error by experimental measurement of ε'' (about 10%). By this the maximum values ε'' and a are risen at $\omega\tau_0 = 1$ and equals accordingly:

$$\varepsilon''_m = \frac{(\varepsilon_s-\varepsilon_\infty)\cos(\beta\pi/2)}{2(1+\sin(\beta\pi/2))}, \tag{7}$$

$$\alpha_m = \frac{\omega(\varepsilon_s-\varepsilon_\infty)}{C_R\sqrt{2(\varepsilon_s+\varepsilon_\infty)}} \cdot \operatorname{tg}\left[\frac{(1-\beta)\pi}{4}\right]. \tag{8}$$

By $\beta = 0$ the equation 8 becomes the following view:

$$\alpha_m = \frac{\omega(\varepsilon_s-\varepsilon_\infty)}{C_R\sqrt{2(\varepsilon_s+\varepsilon_\infty)}}. \tag{9}$$

The frequency, by which the maximum value of absorption coefficient corresponded to intended temperature T_α is determined from condition $\omega_m \cdot \tau(T_\alpha) = 1$ whence it follows:

$$\omega_m = \frac{1}{\tau_\infty} \cdot (-T_\tau/T_\alpha). \tag{10}$$

Thus one can always to obtain, by select the frequency of electromagnetic radiation, to the peak of absorption coefficient was corresponded to intended temperature T_α. The height and the width of this peak are determined by physical parameters of medium.

One can to find the height of the peak by substitution of (10) in (9):

$$\alpha_m = \frac{(\varepsilon_s-\varepsilon_\infty)}{C_R\sqrt{2(\varepsilon_s+\varepsilon_\infty)}} \cdot \operatorname{tg}\left[\frac{(1-\beta)\pi}{4}\right] \cdot \frac{1}{\tau_\infty} \exp\left(-\frac{T_\tau}{T_{\ \alpha}}\right), \tag{11}$$

Analogously one can to find the width of the peak ΔT from condition $\alpha = \alpha_m/2$:

$$\Delta T = \frac{T_\tau \cdot \ln(y_2/y_1)}{(\ln y_1 + T_\tau/T_\alpha)\cdot(\ln y_2 + T_\tau/T_\alpha)}, \tag{12}$$

where

$$y_{1,2} = \left[\left(2 + \sin\frac{\beta\pi}{2} \right) \pm \sqrt{ \left(2 + \sin\frac{\beta\pi}{2}^2 - 1 \right)} \right]^{\frac{1}{1-\beta}} \tag{13}$$

In particular, at $\beta = 0$:

$$y_1 = 2 - \sqrt{3} \approx 0.3\,; \qquad y_2 = 2 + \sqrt{3} \approx 3.7\,.$$

In respect that usually $T_\tau >> T_\alpha$, one can to simplify the equation (12):

$$\Delta T \approx \frac{T_\alpha^2}{T_\tau} \cdot \frac{2\ln(4+\beta\pi)}{1-\beta}. \tag{14}$$

In particular, at $\beta = 0$:

$$\Delta T \approx 2.77 \cdot \frac{T_\alpha^2}{T_\tau}. \tag{15}$$

These formulae are bulky and uncomfortable not only for analytic research, but for numerical simulation. Therefore it is desirable to substitute their on the more simple approximate formulae. The break-line approximation is the simplest. By this the peak $\alpha(T)$ have a view of isosceles triangle; its height is α_m, and its base is $2\Delta T$. Outside peak the absorption coefficient is taken as constant, that equals to α_0. The formula, which determines this approximation, can be written in the following view:

$$\alpha(T) = \begin{cases} \alpha_0 \;\; (at\;\; T < T_\alpha - \Delta T) \\ \alpha_0 + \dfrac{\alpha_m - \alpha_o}{\Delta T} \cdot (T - T_\alpha - \Delta T) \;\; (at\;\; T_\alpha - \Delta T < T < T_\alpha) \\ \alpha_0 + \dfrac{\alpha_m - \alpha_o}{\Delta T} \cdot (T_\alpha + \Delta T - T) \;\; (at\;\; T_\alpha < T < T_\alpha + \Delta T) \\ \alpha_0 \;\; (at\;\; T < T_\alpha - \Delta T) \end{cases} \tag{16}$$

Parameter α_0 is taken as α at the temperature $T_a + 2\Delta T_\alpha$.

2 The model and the set of equations

The space between two coaxial metallic circular pipes (R_1 — the inner pipe radius, R_2 — the outer pipe radius) is filled by easily melted dielectric, that formatted the plug, which length is H. It is, for example, the space between the outer column and the compressor-pump tube of a well, filled by paraffin, frozen high-viscosity oil, ice, gas-hydrate etc. The electromagnetic radiation is directed along this coaxial line to eliminate the plug. When electromagnetic radiation meets a plug on its way, it heats that to melting temperature and therefore eliminates the plug.

The substance of the plug is taken as homogeneous and its physical parameters (density ρ, heat capacity c, heat conductivity λ) are taken as constant but different for solid and liquid. The heat exchange with surrounding medium is taken account on the lateral surface (with the intended coefficient of heat exchange κ), but the heat exchange on the inner surface is taken as neglected.

The characteristic of the plug length H is $\sim$ 100m; of the outer pipe radius R_2 is $\sim$ 0.05m, of the inner pipe radius R_1 is 0.01m. As $H >> R_2$, than it is reasonably to use the one-dimensional heat-transfer equation for description of the heating process, but it is necessary to introduce the drain of heat in this equation to take account the heat exchange with surrounding medium. In terms of assumptions made above this equation can be written in the following view:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}(\lambda \frac{\partial T}{\partial z}) + \alpha \cdot W - b\,(T - T_0), \tag{17}$$

where $b = 2kR_2/(R_2^2 - R_1^2)$ is heat exchange coefficient on the lateral surface with surrounding medium; T_0 — the temperature of the surrounding medium; W — density of electromagnetic radiation power.

The physical parameters are intended as follows:

$$\rho = \begin{cases} \rho_0 & \text{at} \quad T \leq T_s \\ \rho_1 & \text{at} \quad T > T_s \end{cases} \tag{18}$$

$$\lambda = \begin{cases} \lambda_0 & \text{at} \quad T \leq T_s \\ \lambda_1 & \text{at} \quad T > T_s \end{cases} \tag{19}$$

$$c = \begin{cases} c_0 \quad \text{at} \quad T < T_s \\ L\delta(T - T_s) \quad \text{at} \quad T = T_s \\ c_1 \quad \text{at} \quad T > T_s \end{cases} \tag{20}$$

where T_s, L — are the temperature and the heat of transition (melting) accordingly; $\delta(T - T_s)$ — delta-function, that was replaced by numerical simulation on the delta-similar function — "footstep" with the limited width $2\Delta T_s$:

$$c(T) = \begin{cases} c_0 \quad \text{at} \quad T < T_s - \Delta T_s \\ \dfrac{c_0 + c_1}{2} + \dfrac{L}{2\Delta T_s} \quad \text{at} \quad T_s - \Delta T_s < T < T_s + \Delta T_s \\ c_1 \quad \text{at} \quad T > T_s + \Delta T_s \end{cases} \tag{21}$$

The density of electromagnetic radiation power W is:

$$W = W_0 \cdot \exp\{-\int\limits_0^z \alpha(z', t)\, dz'\} = W_0 \cdot e^{-\Gamma(z,t)}, \tag{22}$$

where $W_0 = P/[\pi(R_2^2 - R_1^2)]G$ — the density of electromagnetic radiation power on the top surface of plug,

$$\Gamma(z, t) = \int\limits_0^z \alpha(z', t)\, dz' \tag{23}$$

— the integral absorption coefficient. If one can consider the absorption coefficient $\alpha(T)$ as constant, that $\Gamma = \alpha z$, and formula (22) is reduced to Lambert law of absorption:

$$W = W_0 \cdot \exp(-\alpha z). \tag{24}$$

The heat exchange on the top and bottom of plug is taken as neglected, and boundary conditions to equation (17) are:

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = \left.\frac{\partial T}{\partial z}\right|_{z=H} = 0. \tag{25}$$

It is comfortable to write these equations in the measureless form: Let it be the following designations: $\tau = \lambda_0 t/(c_0\rho_0 H^2) = a_0 t/H^2$ — the measureless time, $\theta = (T - T_0)/(T_s - T_0)$ — the measureless temperature, $x = z/H$ — the measureless coordinate, $\Phi = c\rho/(c_0\rho_0)$ — the measureless volume heat capacity, $\Lambda = \lambda/\lambda_0$ — the measureless heat conductivity, $J = L/[c_0(T_s - T_0)]$ — the measureless heat of melting, $Q_0 = H \cdot W_0/[\lambda_0(T_s - T_0)]$ — the measureless density of electromagnetic radiation power on the top surface of plug, $B = b \cdot H^2/\lambda_0$ — the measureless heat exchange coefficient on the lateral surface with surrounding medium, $\gamma = H \cdot \alpha$ — the measureless absorption coefficient of electromagnetic radiation.

In these designations equation (17) becomes the following view:

$$\Phi\frac{\partial\theta}{\partial\tau} = \frac{\partial}{\partial x}\left(\Lambda\frac{\partial\theta}{\partial x}\right) + \gamma Q_0 e^{-\Gamma} = 0. \tag{26}$$

where

$$\Gamma = \int\limits_0^z \alpha dz' = \int\limits_0^z \gamma x' \tag{27}$$

The formula (16) also can be written in more comfortable view. Will be:

$$\theta_\alpha = \frac{T_\alpha - T_0}{T_s - T_0}\,, \qquad \Delta\theta_\alpha = \frac{\Delta T_\alpha}{T_s - T_0}\,,$$

$$\gamma_0 = H \cdot \alpha_0\,, \qquad \gamma_m = H \cdot \alpha_m\,,$$

$$\gamma_1 = \gamma_0 + k(\theta_\alpha + \Delta\theta_\alpha)\,,$$

$$\gamma_2 = \gamma_0 - k(\theta_\alpha - \Delta\theta_\alpha)\,, \qquad k = \frac{\gamma_m - \gamma_0}{\Delta\theta_\alpha}\,.$$

Then the formula (16) can be written in view:

$$\gamma(\theta) = \begin{cases} \gamma_0 & \text{at} \quad \theta \geq \theta_\alpha + \Delta\theta_\alpha\,, \\ \gamma_1 - k\theta & \text{at} \quad \theta_\alpha \leq \theta \leq \theta_\alpha + \Delta\theta_\alpha\,, \\ \gamma_2 + k\theta & \text{at} \quad \theta_\alpha - \Delta\theta_\alpha \leq \theta \leq \theta_\alpha\,, \\ \gamma_0 & \text{at} \quad \theta \leq \theta_\alpha - \Delta\theta_\alpha\,. \end{cases} \tag{28}$$

3 The stationary analytical solution

For the steady-state temperature $\partial\theta/\partial t = 0$, and the equation (17) becomes the view:

$$\frac{d}{dx}\left(\Lambda\frac{d\theta}{dx}\right) + \gamma Q_0 e^{-\Gamma} - B\theta = 0, \tag{29}$$

For our case the values of $B >> 1$, $Q_0 >> 1$, $d\theta/dx \sim\ v1$, $\Lambda \sim 1B$ are characteristic, therefore in the equation (29) the first term is neglected. It means physically, that the heat transfer due to conductivity is neglected in comparison with volume heating due to absorption of electromagnetic radiation and with the heat exchange with surrounding medium. The equation (29) becomes in this case the view:

$$\gamma Q_0 e^{-\Gamma} - B\theta = 0. \tag{30}$$

3.1. The monotone decreasing steady-state temperature field

If initial temperature, as usually, is constant, than the monotone decreasing steady-state temperature field will set after long-time heating: the highest temperature will be at top (at $x = 0$), and the least temperature will be at bottom (at $x = 1$).

If the power of source and the other parameters are such, so $\theta(0) > \theta_\alpha + \Delta\theta_\alpha$ and $\theta(1) < \theta_\alpha - \Delta\theta_\alpha$, that one may to eliminate 4 regions in all range $0 \leq x \leq 1$:

1) $0 \le x \le x_1$, where $\theta \ge \theta_\alpha + \Delta\theta_\alpha$;
2) $x_1 \le x \le x_\alpha$, where $\theta_\alpha + \Delta\theta_\alpha \ge \theta \ge \theta_\alpha$;
3) $x_\alpha \le x \le x_2$, where $\theta_\alpha \ge \theta \ge \theta_\alpha - \Delta\theta_\alpha$;
4) $x_2 \le x \le 1$, where $\theta \le \theta_\alpha - \Delta\theta_\alpha$.

In the region 1:

$$\theta = \gamma_0 \cdot \frac{Q_0}{B} \cdot e^{-\gamma_0 x}. \tag{31}$$

In particular, at $x = x_1$:

$$\theta = \theta_\alpha + \Delta\theta_\alpha = \gamma_0 \frac{Q_0}{B} e^{-\gamma_0 x_1} = \gamma_0 \frac{Q_1}{B}\,, \tag{32}$$

where

$$Q_1 = Q_0 e^{-\gamma_0 x_1} = \frac{B}{\gamma_0}(\theta_\alpha + \Delta\theta_\alpha)\,. \tag{33}$$

In the region 2:

$$\gamma = \gamma_1 - k\theta. \tag{34}$$

Let it be

$$\Gamma_1(x) = \int\limits_{x_1}^{x} (\gamma_1 - k\theta) dx'; \tag{35}$$

than the equation (30) becomes:

$$\gamma Q_1 e^{-\Gamma_1(x)} - B\theta = 0. \tag{36}$$

Taken θ from (34) and take account, that according to (35), $\gamma = \Gamma_1'$, one received differential equation for function $\Gamma_1(x)$:

$$\frac{d\Gamma_1}{dx} Q_1 e^{-\Gamma_1(x)} - \frac{B}{k}\left(\gamma_1 - \frac{d\Gamma_1}{dx}\right) = 0. \tag{37}$$

Taken the integral from x_1 to x, one found:

$$Q_1\left(1 - e^{-\Gamma_1(x)}\right) + \frac{B}{k}\Gamma_1(x) = \frac{B}{k}\gamma_1(x - x_1), \tag{38}$$

or, take account (33),

$$\frac{\theta_\alpha + \Delta\theta_\alpha}{\gamma_0}\left(1 - e^{-\Gamma_1(x)}\right) + \frac{1}{k}\Gamma_1(x) = \frac{1}{k}\gamma_1(x - x_1). \tag{39}$$

Equation (38) (or (39)) described fully the function $\Gamma_1(x)$.

The dependence $\theta(x)$ can be find from (30) or (34):

$$\theta = \frac{\gamma_1 Q_1 e^{-\Gamma_1(x)}}{B + Q_1 k e^{-\Gamma_1(x)}} = \frac{\gamma_1(\theta_\alpha + \Delta\theta_\alpha)e^{-\Gamma_1(x)}}{\gamma_0 + k(\theta_\alpha + \Delta\theta_\alpha)e^{-\Gamma_1(x)}}. \tag{40}$$

Taken $\theta = \theta_\alpha$, one find $\Gamma_1(x_\alpha)$:

$$\begin{aligned} e^{-\Gamma_1(x_\alpha)} &= \frac{\gamma_0\theta_\alpha}{\gamma_m(\theta_\alpha + \Delta\theta_\alpha)}, \\ \Gamma_1(x_\alpha) &= \ln\frac{\gamma_m(\theta_\alpha + \Delta\theta_\alpha)}{\gamma_0\theta_\alpha} \end{aligned} \tag{41}$$

because the point, in which the temperature is $q = q_\alpha$, equals:

$$x_\alpha = x_1 + \frac{k}{\gamma_1}\left(\frac{\theta_\alpha + \Delta\theta_\alpha}{\gamma_0} - \frac{\theta_\alpha}{\gamma_m}\right) + \frac{1}{\gamma_1}\ln\frac{\gamma_m(\theta_\alpha + \Delta\theta_\alpha)}{\gamma_0\theta_\alpha}. \tag{42}$$

In the region 3:

$$\theta = \frac{\gamma_2\theta_\alpha e^{-\Gamma_2(x)}}{\gamma_m - k\theta_\alpha e^{-\Gamma_2(x)}}, \tag{43}$$

where

$$\Gamma_2(x) = \int\limits_{x_\alpha}^{x}(\gamma_2 + k\theta)dx'. \tag{44}$$

The function $\Gamma_2(x)$ is directed by formula:

$$-\frac{\theta_\alpha}{\gamma_m}\left(1 - e^{-\Gamma_2(x)}\right) + \frac{1}{k}\Gamma_2(x) = \frac{\gamma_2}{k}(x - x_\alpha). \tag{45}$$

In particular, at $x = x_2$ the temperature is $\theta = \theta_\alpha - \Delta\theta_\alpha$, and

$$\begin{aligned} e^{-\Gamma_2(x_2)} &= \frac{\gamma_m(\theta_\alpha - \Delta\theta_\alpha)}{\gamma_0\theta_\alpha}, \\ \Gamma_2(x_2) &= \ln\frac{\gamma_0\theta_\alpha}{\gamma_m(\theta_\alpha + \Delta\theta_\alpha)}, \end{aligned} \tag{46}$$

and the coordinate of point x_2 equals:

$$x_2 = x_\alpha + \frac{k}{\gamma_2}\left(\frac{\theta_\alpha - \Delta\theta_\alpha}{\gamma_0} - \frac{\theta_\alpha}{\gamma_m}\right) - \frac{1}{\gamma_2}\ln\frac{\gamma_m(\theta_\alpha - \Delta\theta_\alpha)}{\gamma_0\theta_\alpha}. \tag{47}$$

In the region 4:

$$\theta = (\theta_\alpha - \Delta\theta_\alpha)e^{-\gamma_0(x-x_2)}. \tag{48}$$

3.2. The monotone increasing steady-state temperature field

In temperature range of $\theta_\alpha - \Delta\theta_\alpha \leq \theta \leq \theta_\alpha$ the equation (30) have the solution, for that the temperature increases with the growth of coordinate x. That is the top of plug is heated weakly than the bottom in spite of the top is nearly to the source of radiation. It is possible only for volume heating at well speed increasing of absorption coefficient with the growth of the temperature. By that the radiation goes through weakly heating region almost without absorption and is absorbed strongly in region, that have the more high temperature.

As it showed by analytic and numerical researches, this solution is unstable. At the small decreasing of the power the "overheated" zone disappears, and at small increasing of the power that begin to move to initial of coordinates ("the reverse temperature wave"). In both cases the stable steady-state solution with increasing temperature sets.

If the power os the source is

$$Q_0 = \frac{B}{\gamma_0}(\theta_\alpha - \Delta\theta_\alpha). \tag{49}$$

than the temperature on top of the plug equals to $\theta_\alpha - \Delta\theta_\alpha$.

If one received on someway on range $0 \leq x \leq x_\alpha$ the increasing temperature, such as $\theta(x_\alpha) = \theta_\alpha$, then the equation (30) becomes the view:

$$\frac{\theta_\alpha - \Delta\theta_\alpha}{\gamma_0}\frac{d\Gamma}{dx}e^{-\Gamma(x)} + \frac{1}{k}\left(\gamma_2 - \frac{d\Gamma}{dx}\right) = 0, \tag{50}$$

where

$$\Gamma(x) = \int\limits_0^x (\gamma_2 + k\theta)dx'. \tag{51}$$

By integration of this equation one finds:

$$\frac{\theta_\alpha - \Delta\theta_\alpha}{\gamma_0}\left(1 - e^{-\Gamma(x)}\right) - \frac{1}{k}\Gamma(x) = -\frac{\gamma_2}{k}x, \tag{52}$$

and

$$\theta = \frac{\gamma_2(\theta_\alpha - \Delta\theta_\alpha)e^{-\Gamma(x)}}{\gamma_0 - k(\theta_\alpha - \Delta\theta_\alpha)e^{-\Gamma(x)}}. \tag{53}$$

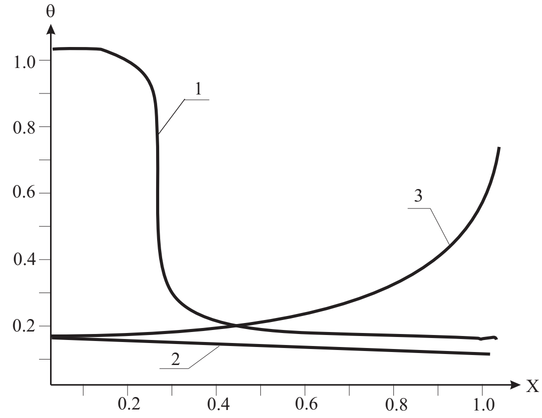

Figure 1: Steady-state temperature fields

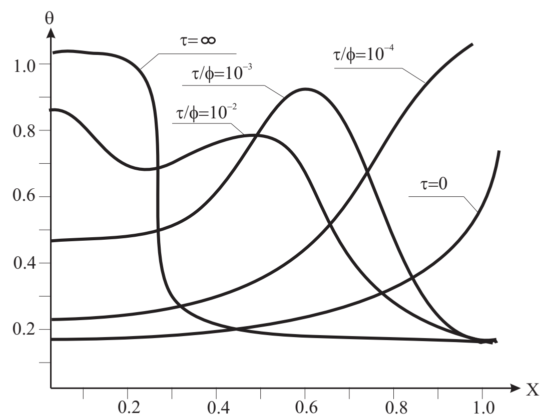

Figure 2: Steady-state temperature fields

For the temperature was be increasing, the derivation $d\theta/dx$ must to be positive:

$$k(\theta_\alpha - \Delta\theta_\alpha)e^{-\Gamma} > \gamma_0. \tag{54}$$

In particular, at $x = 0$ the derivation $d\theta/dx$ is:

$$\theta'(0) = \frac{\gamma_0^2(\theta_\alpha - \Delta\theta_\alpha)}{-\gamma_2}. \tag{55}$$

i.e. $d\theta/dx > 0$ if $\gamma_2 < 0$.

On the Fig. 1 are showed three steady-state temperature fields, which (in dependence on initial conditions) may be received in the same medium at the same power of radiation. Each of these fields correspondes to some solution of equation (30); two from their (curves 1 and 2) correspond to formulae (48) and (40), and one (curve 3) correspondes to formula (53).

The Fig. 2 showes the results of numerical experiment: the increasing temperature field turns into "reverse temperature wave" after short-times decreasing of heat exchange coefficient. In finally the corresponding to curve 2 (Fig. 1) field sets.

Conclusions

The efficiency of high-frequency electromagnetic heating depends on the physical parameters of medium and on frequency and power of radiation. Using the nonlinear properties of medium, one can to rise substantially the efficiency of the heating and to receive effects, those are impossible by usual conditions.

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